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Evaluation of Multivariable Regression in Predicting Rock Slake Durability Index

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ABSTRACT

The durability of stone against erosion, weathering and dissolution against water flow is an important parameter in Civil engineering projects. It is absolutely necessary to check the quality of slake durability rock against water flow in all types of stone breakwaters, rips - rap in dams, including building materials (a wide range of decorative stones). Slake durability testing is a common method for assessing the durability of rocks. The durability of stone has a lot to do with the texture, weathering and physical and mechanical properties of the stone. In this study, by performing slake durability tests on samples belonging to the Tale Zang Formation, the slake durability index (SDI) of limestone belonging to this formation was investigated. The effect of the number of cycles and the relationship between slake durability index and physical and mechanical properties have also been investigated. Univariable and multivariable regression relationships were used to predict the slake durability index. The results show that multivariable linear regression using a combination of geomechanical properties offers a higher correlation between these properties and the slake durability index (SDI) of the studied limestones. In multivariable regression models, n, UCS, E and BTS properties in different relationships have the greatest effect on the SDI value

Keywords: Correlation, Geomechanical properties, Havasan dam, Linear relation, Residual, Statistical analysis

Introduction

One of the most important geotechnical hazards at the site of hydraulic structures such as dams is the Slake of rock masses due to successive wetting and drying. As a result of this behavior, rock masses become fine fragments and soil [1]. The Slake process occurs in the form of particle dissolution, cracking and scaling of surface layers [2]. Slake changes the physical and mechanical properties of rocks and reduces their bearing capacity. Therefore, before designing and constructing civil projects, it is necessary to determine the amount of changes resulting from this process.
The sensitivity of rock to weathering and the rate of weathering occurs is measured quantitatively by a property called the Slake Durability index (SDI). This feature is used in the modified geomechanical classification (M-RMR). The significant advantages of this test can be called the lack of sampling and the very low cost of the test compared with the other tests. One of the most common methods for measuring rock weathering, Slake Durability Index (SDI), which by Franklin and Chandra in [3]. In 1979 by the International Society for Rock Mechanics as the right way to conduct test Slake Durability Index was proposed [4]. This test is used to determine the strength of a rock sample affected by two stages of successive wetting and drying. The percentage of weight remaining in the sample after these two stages of wetting and drying is called the slake durability index, which indicates the degree of stability of the rock against natural weathering. The higher the value of this index, the lower the rate of erosion, dissolution and crushing of the rock against weathering. Determination of SDI is required for rocks that are in the abutment, body and around dams and rock materials used in the construction of breakwaters.
Many studies have been performed for the durability of different rocks using the SDI. Yagiz [5] performed a study to assess the relationships between the SDI and

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uniaxial compressive strength (UCS), Schmidt hardness, P wave velocity, modulus of elasticity, effective porosity, water absorption and dry and saturated unit weights. Fereidooni and Khajevand [6] performed the Physical and mechanical properties of the samples mean dry and saturated unit weights, specific gravity, porosity, water absorption, ultrasonic P wave velocity, Schmidt rebound hardness, point load index, Brazilian tensile strength, block punch index, uniaxial compressive strength and slake durability tests on Travertine rocks to correlate the mentioned rock properties. They used univariable regression to determine the relationship between the SDI and engineering properties. Ahmad et al. [7] performed the Strength indices such as point load index, impact strength index, aggregate impact value, aggregate crushing value and durability indices such as slake durability index, aggregate abrasion value on different rocks to correlate the mentioned rock properties. They used univariable and multivariable regression to determine the relationship between slake durability index and strength and durability indices. In the study of Abd El Aal and Kahraman [8], the predictability of the slake durability (SDI) of marbles were investigated from the indirect methods such as point load index (IS50), Shore hardness index (SHI), impact strength index (ISI), and ultrasonic velocity (Vp). The evaluation of the test results showed that the SDI were correlated well to the Vp, and moderately correlated to the IS50. But, the SDI was correlated weakly to both the ISI and SHI values. Some multiple regression equations were also derived for the estimation of the SDI. In the study of Bozdag and Ince [9], a very high exponential relation (univariable regression) between the strength parameters (point load strength, uniaxial compressive strength and Brazilian tensile strength) and SDI was found out. Zorlu and Yagiz [10] performed a study to assess the relationships between results of slake durability test and fractal dimension of aggregates. In the study of Seyed Mousavi et al. [11], the influence of freezing-thawing cycles on durability index (Cycles 2 and 5) has been evaluated. The obtained results showed that durability index decreased exponentially. Aksoy et al. [12] performed the correlation of spherical samples with slake durability index for different rock types. In the study of Monticelli et al. [13] presents a study on durability index obtained in the slake durability test (SDT) and the uniaxial compressive strength (UCS) of a gneissic rock from the Nova Venécia Complex, southeastern Brazil. In the study of Beyhan et al. (14), the effect of pH of the test fluid on dispersibility of rocks was evaluated by performing slake durability index test on two different marl rock units in acidic, neutral and alkaline environments. In the study of NAVARRETE Seras et al. (2020), the slake durability index (Cycle 2), density, and absorption data were correlated to obtain a mathematical model that helps predict Slake Durability index (Cycle 2) and verify the relationship between the variables. On the other hand, logistic regression was used to classify rock quarries according to their durability index. Arman [15], performed the correlation of uniaxial compressive strength with 2nd cycle of slake durability index for evaporitic rocks. The relations between UCS and SDI-2st were estimated with representative univariable empirical equations.

Several factors affect the SDI value and considering these factors in predicting the SDI value can provide both statistically and geotechnically more reliable results For this purpose, using multivariable regression is a simple and valid method. The use of different geomechanical parameters in determining SDI has received less attention in previous studies. In some studies, quantitative parameters have been used to predict SDI. In addition, no study has examined the degree of increase in the accuracy of the results when using multivariable regression. In this study, SDI prediction was performed using different geomechanical parameters and univariable and multivariable regression methods. In this study, experiments performed on samples of Havasan dam rock (Northwest of Iran) have been used.

Havasan Dam

Havasan Reservoir Dam is a pebble dam with a clay core with a height of 54 m from the foundation and a crown length of 1200 m and a reservoir volume of 70 million cubic meters to supply agricultural water and generate electricity on the Havasan River approximately 39 km northwest of Sar-e-pol-e Zahab. It is one of the functions of Kermanshah province and is located in the northwest of Iran. Fig. 1 shows the location of the Havasan Dam.

The stratigraphic units that make up the site of the Havasan Dam include calcareous and shale marl rock units of the late second and early third periods, as well as young deposits. The left abutment and bedrock of the Havasan Dam form the Tale Zang Formation, which consists of light gray to sandy to marly limestones and sometimes crystalline. The Tale Zang Formation is a unit of Paleocene to Middle Eocene carbonates in the pattern section and consists of 870 m of gray limestone, which is typically located on detrital units of the Amiran Formation and below the Kashkan Formation [16]. The right abutment of the dam is formed by the Shaly and Marly Amiran Formation, which is often composed of alternating Marl and dark gray Shale layers. Fig. 2 shows the three-dimensional model of the dam and the location of the Tale Zang and Amiran Formations in the Hoasan Dam site. Fig. 3 also shows the geological section of the dam axis and the position of the drilled boreholes.

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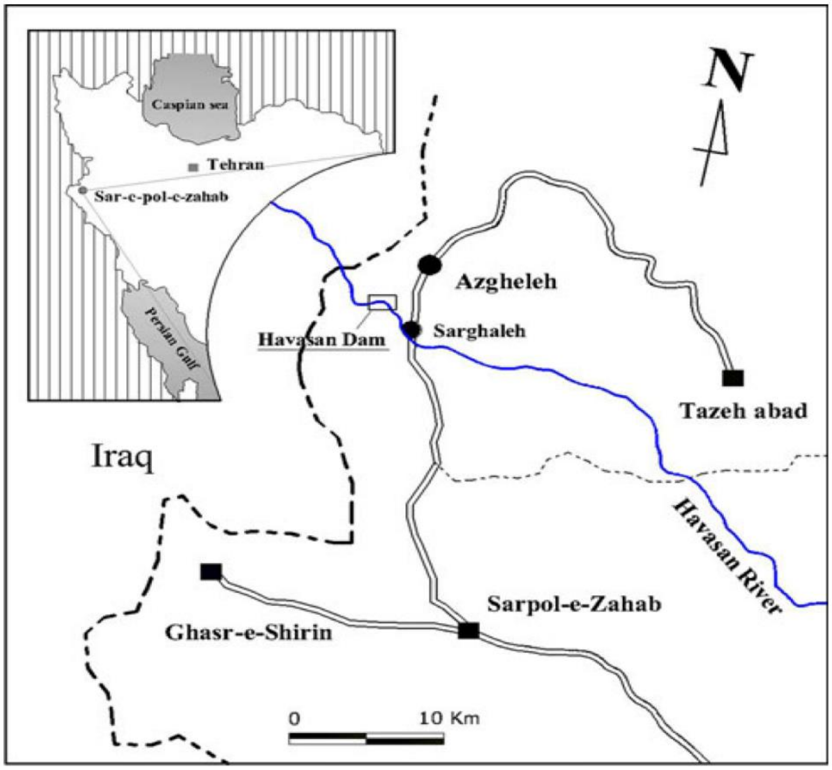

Figure 1. Location of the Havasan Dam in Iran

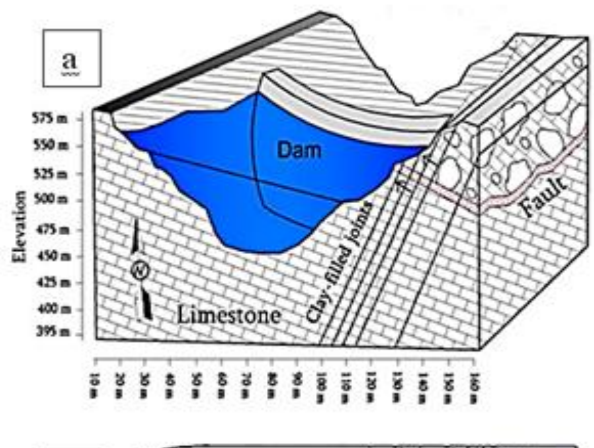

Figure 2. a) Three-dimensional situation of Havasan Dam, b) Location of Tale Zang and Amiran Formations in Havasan Dam construction site [17]

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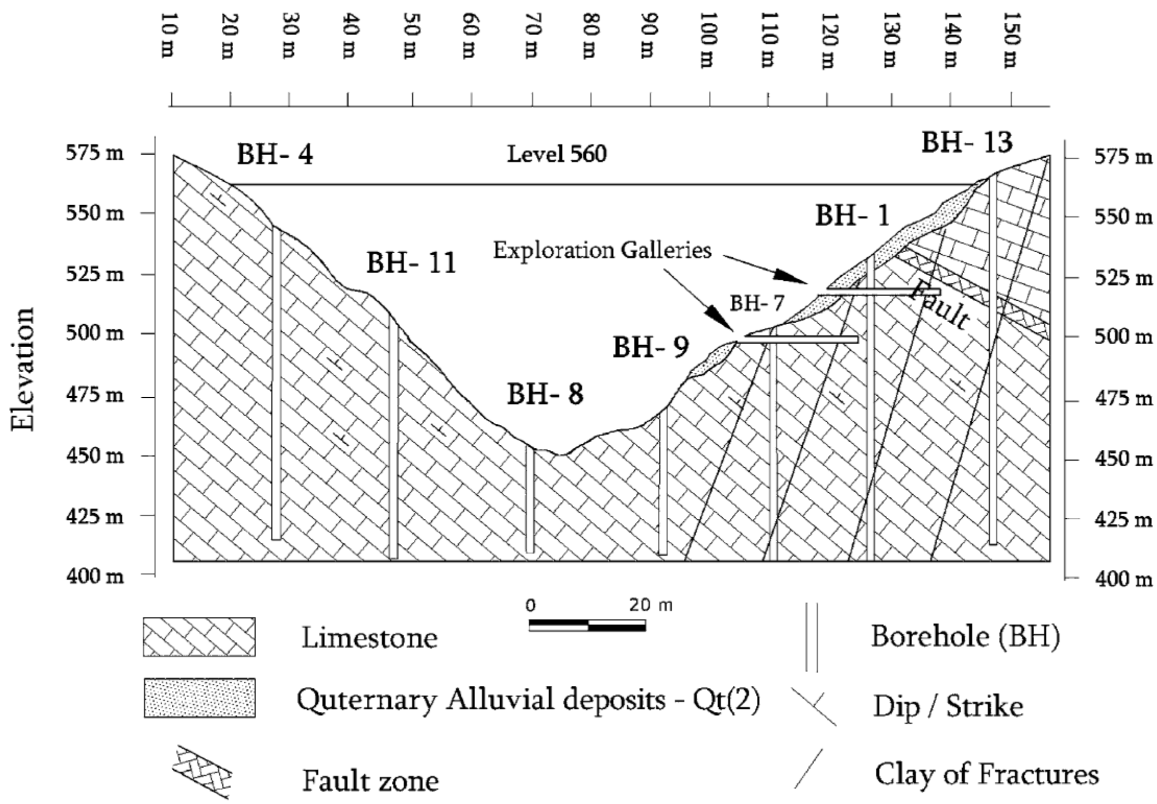

Figure 3. Geological cross section and location of boreholes along the dam axis [17]

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Materials and Methods

To carry out the study, Limestone samples were selected from Havasan Dam in northwest of Iran. Among the most important physical properties of rock mass in geotechnical studies include Porosity (n), Dry density (ρ_{dry}), Saturation Density (ρ_{sat}) and Moisture Content (W%) that have been used in this study. These properties are performed according to ISRM [18] standards. The mechanical properties used in this study include Poisson's ratio (υ_{sat}), uniaxial compressive strength (UCS_{sat}), modulus of elasticity (E_{sat}), Brazilian tensile strength (BTS_{sat}), P wave velocity (V_{p-dry} and V_{p-sat}) and S wave velocity (V_{s-dry} and V_{s-sat}). The BTS was used to evaluate the tensile strength. This test was performed according to ASTM D 3967 standard [19]. To determine this parameter, samples with a diameter of 54 mm and a thickness of 24 to 31 mm were prepared. The parameters of UCS, E and υ were also estimated according to ASTM D7012 standard [20]. To perform the uniaxial compression test (UCS), cores with a diameter of 54 mm and a length of 110 to 130 mm were used and the two ends of each sample were cut using a perfectly smooth and parallel saw. Vp and Vs were determined according to ASTM D2845 standard [21]. To evaluate the wave velocity, a sample of rocks with a diameter of 54 mm and a length of 25 to 30 was prepared. Also, to evaluate the Slake Durability Index (SDI) and to prepare the stone samples, the test method was performed according to ASTM D4644 standard [22]. The slake durability index (SDI) was calculated as:

$$SDI_i = \frac{W1 - B}{A - B} \times 100 \qquad (1)$$

Where SDI_i is i_{st} cycle slake durability index (%), W1 is weight of drum plus sample retained after i_{st} cycle (gr), A is mass of drum and oven dried sample before i_{st} cycle (gr), B is mass of drum (gr).

Descriptive statistical (Frequency histogram and normal curve) results of physical and mechanical properties of Tale Zang limestone samples are presented in Fig. 4. The SDI for the first cycle is 99% and for the second cycle is 98.17%. The rock mass classification of the Tale Zang Formation based on the SDI according to the Gamble [23], Franklin & Chandra [3] and Zhu and Deny [24] classifications is very high durability. The difference between the dry density (2.49 gr/cm^3) and the saturation density (2.55 gr/cm^3) of the samples is small due to the low porosity of the samples. According to the Anon [25] classification for density and porosity, most samples are in the high (2.55-2.75 gr/cm^3) and medium (2.2-2.55 gr/cm^3) density and low (1-5%) and medium (5-15%) porosity categories.

Moisture content in most samples is less than 5%. According to the ISRM [18] classification, most samples are classified as medium-strength (20-70 MPa) rocks based on UCS. According to the classification of rocks based on Vp by Anon [25], most samples are in the category of medium (3500-4000 m/s) and high (4000-5000 m/s) velocity rocks. As can be seen in Fig. 4, the frequency distribution of the various properties is almost normal.

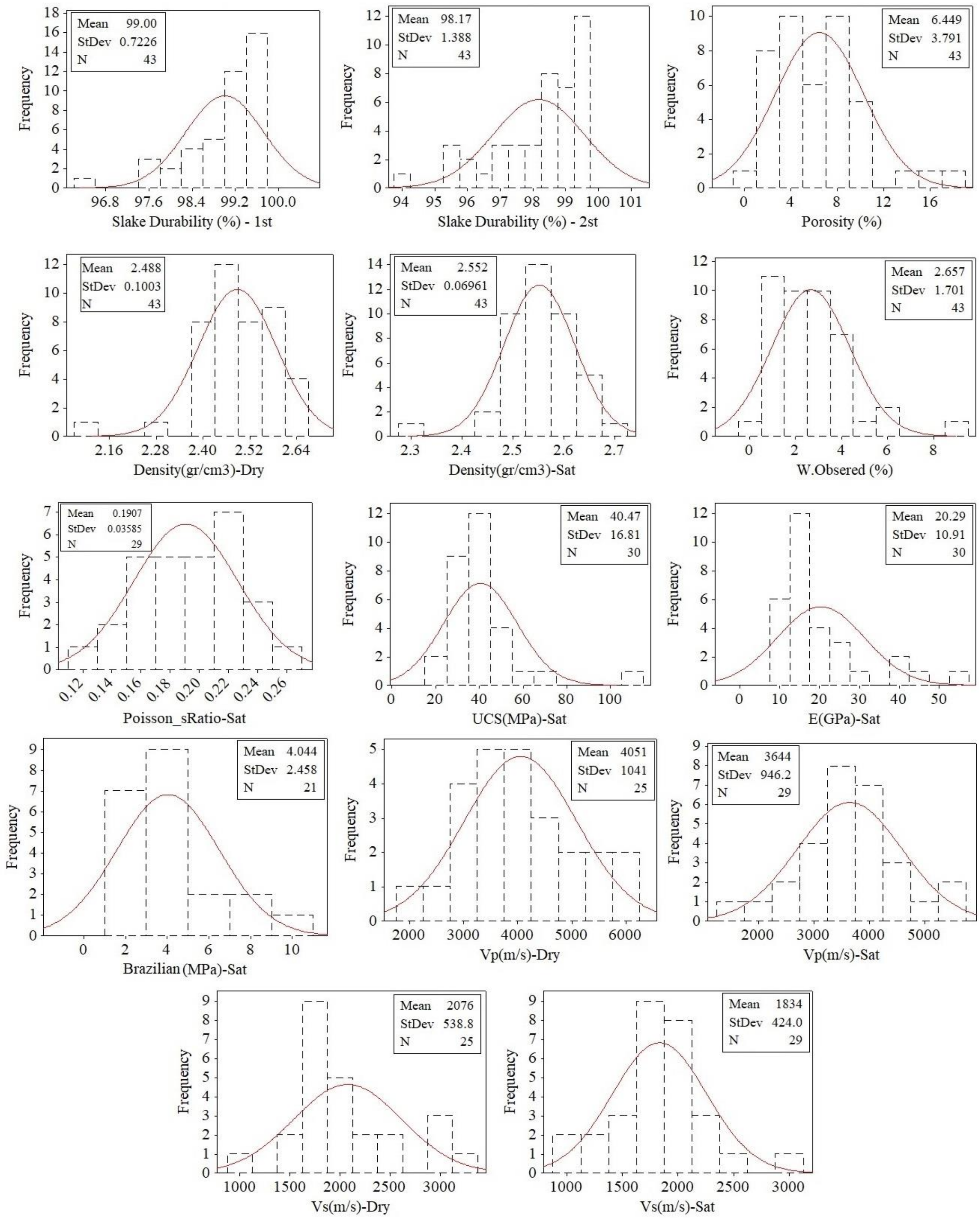

Figure 4. Frequency histogram and normal curve of primary data physical and mechanical properties of drilling cores

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In this study, first, using univariable regression, the relationship between the SDI with physical and mechanical properties was investigated. In the next step, multivariable regression was used to predict the SDI. At each stage, to validate each of the relationships, the residual analysis and the relationship between the measured and predicted values were performed. Finally, in the discussion section, the degree of increase in accuracy in multivariable relationships compared to univariable relationships was investigated.

Results

Prediction of Slake Durability Index (SDI) using univariable regression

Various studies show that regression analysis is a simple and effective method in geotechnical surveys and is widely used today [26, 27, 28, 29]. However, in previous studies, the use of multivariable regression has been less commonly used to predict the SDI.

After deleting outlier from the data, the SDI was predicted using univariable regression. Regression analysis shows that there is no valid relationship ($R^2<0.47$) between the SDI and depth (Fig. 5). However, the second cycle SDI (SDI_{2st}) is more correlated with depth.

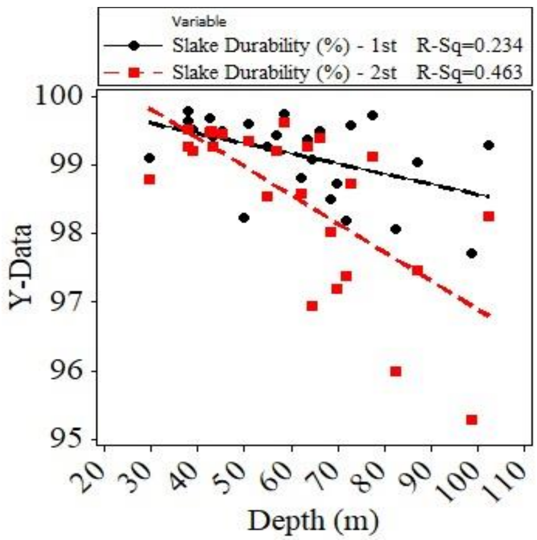

Figure 5. Relationship between Slake Durability Index (SDI) with depth (meters)

The univariable relationship (95% Confidence intervals) between the SDI (cycles 1 and 2) and physical properties is shown in Fig. 6. The relationship between SDI with dry and saturated density is direct and with n and W is inverse. The results of regression analysis are presented in Table (1) which shows that all these relationships are significant (Sig <0.05). According to Table (1), the relationship between SDI_{2st} with W (R^2 = 0.65 and F = 69.26) is the most valid relationship and the relationship between SDI_{1st} with saturation density (R^2 = 0.48 and F = 35.38) is the least valid. The results show that the SDI values of the first cycle are higher than the second cycle (Fig 6).

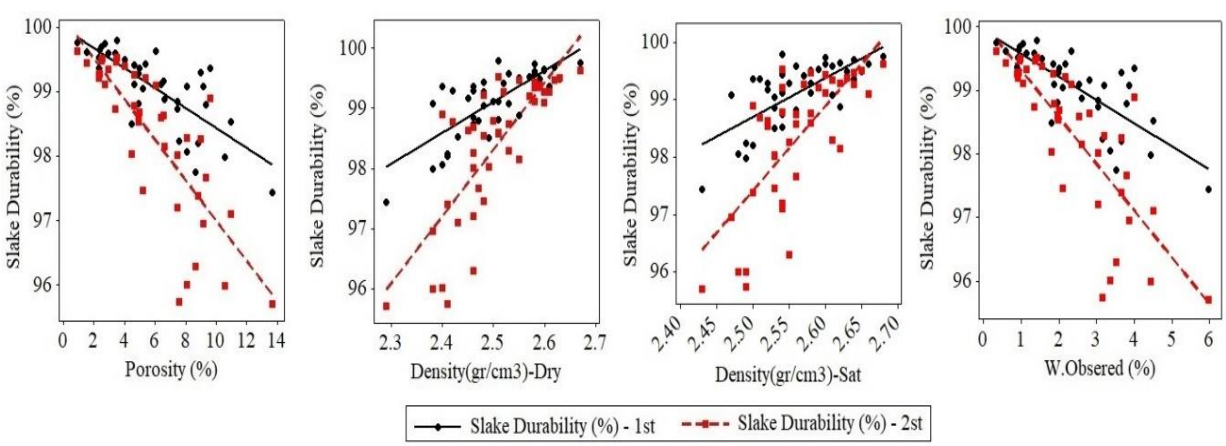

Figure 6. Relationship between SDI with physical properties

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Table 1
Results of univariable regression analysis of physical properties

Regression equation	R square (R^2)	Result of ANOVA test F	Sig.
$SDI_{1st} = 99.98 - 0.15\ n$	0.60	55.43	0.00
$SDI_{2st} = 100.1 - 0.31\ n$	0.64	66.08	0.00
$SDI_{1st} = 86.22 + 5.15\rho_{dry}$	0.59	52.61	0.00
$SDI_{2st} = 70.53 + 11.12\rho_{dry}$	0.62	61.60	0.00
$SDI_{1st} = 81.74 + 6.78\ \rho_{sat}$	0.48	35.38	0.00
$SDI_{2st} = 60.59 + 14.73\ \rho_{sat}$	0.52	41.08	0.00
$SDI_{1st} = 92.94 - 0.37\ W$	0.61	58.09	0.00
$SDI_{2st} = 100.1 - 0.74\ W$	0.65	69.26	0.00

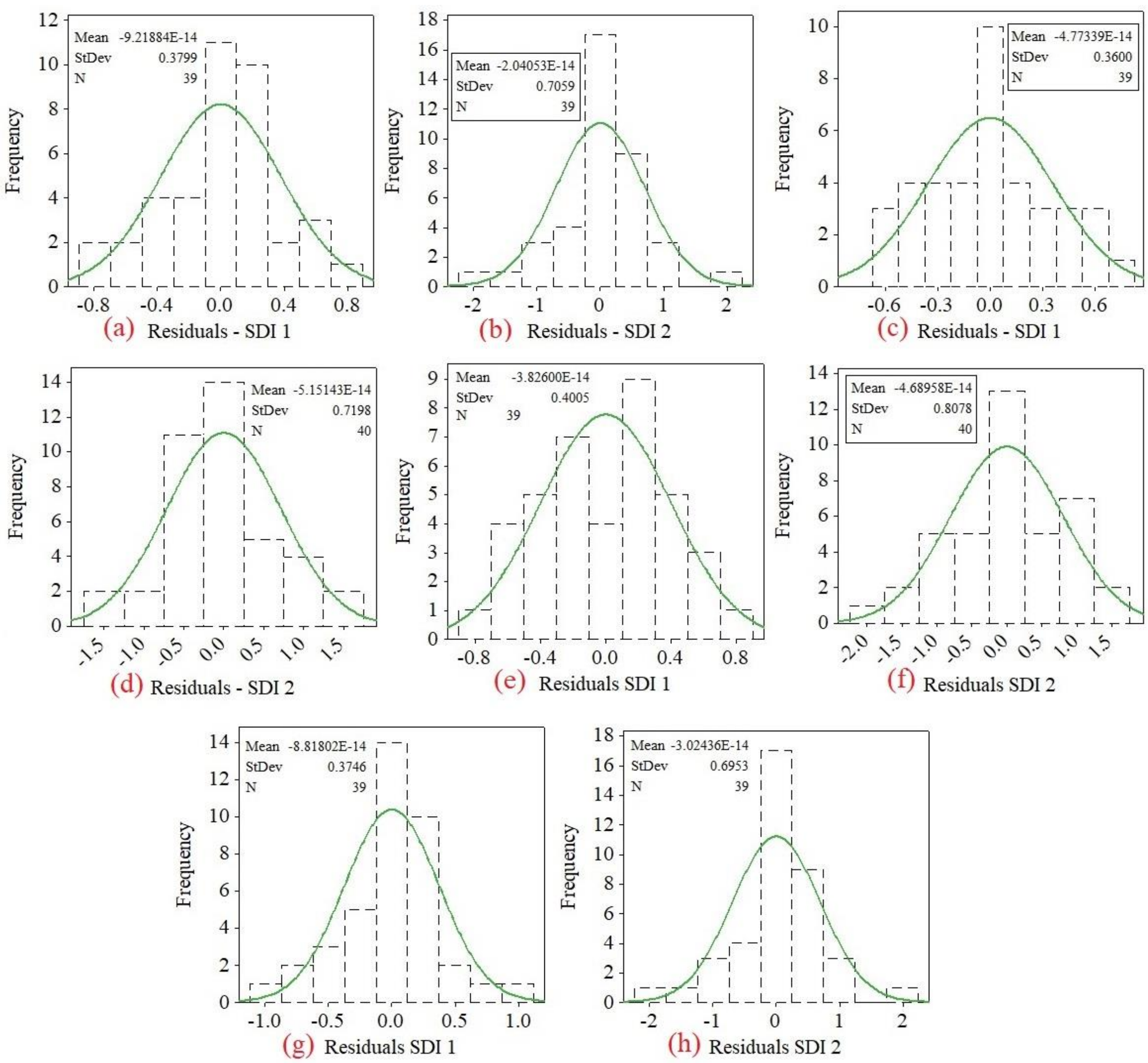

Figure 7. Frequency histogram and normal curve of univariable regression residuals physical properties. Relationship between: (a) SDI-1st with n, (b) SDI-2st with n, (c) SDI-1st with ϱ-dry, (d) SDI-2st with ϱ-dry, (e) SDI-1st with ϱ-sat, (f) SDI-2st with ϱ-sat, (g) SDI-1st with W and (h) SDI-2st with W

One of the valid methods for validation of estimation tools is residuals analysis (difference between measured values and predicted values) which is widely used in geotechnical studies [30, 31, 28, 32, 33, 29). In the residuals analysis, the normality of the residuals distribution and the zero of the residuals mean indicate the high validity of the model. Fig. 7 shows the frequency and statistical description of the residuals of

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univariable regression relations of physical properties. According to Fig. 7 for most relationships, the frequency of residuals is zero. For all relationships, the mean of the residuals tends to zero. Also, the frequency distribution of the residuals is almost normal for all relationships. Therefore, based on the residuals analysis, all relationships are sufficiently valid.

Another widely used validation method is to determine the correlation between measured values and estimated values. The existence of correlation ($R^2 = 1$) between measured and estimated values in linear regression indicates the high validity of the estimated values. Fig. 8 shows the relationship between measured and predicted SDI using physical properties. Based on this Fig., the predicted values using all relations show a correlation of more than 0.889.

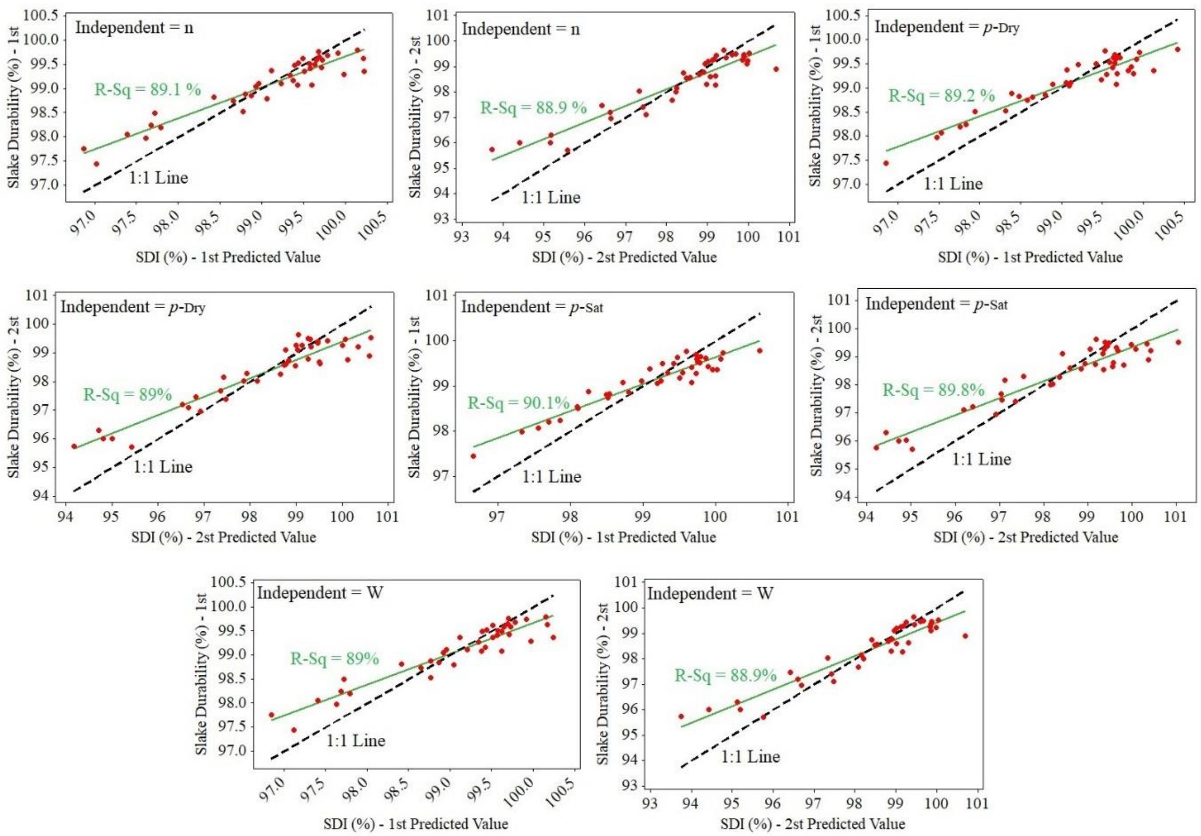

Figure 8. Comparison of measured and predicted SDI values using univariable regression of physical properties

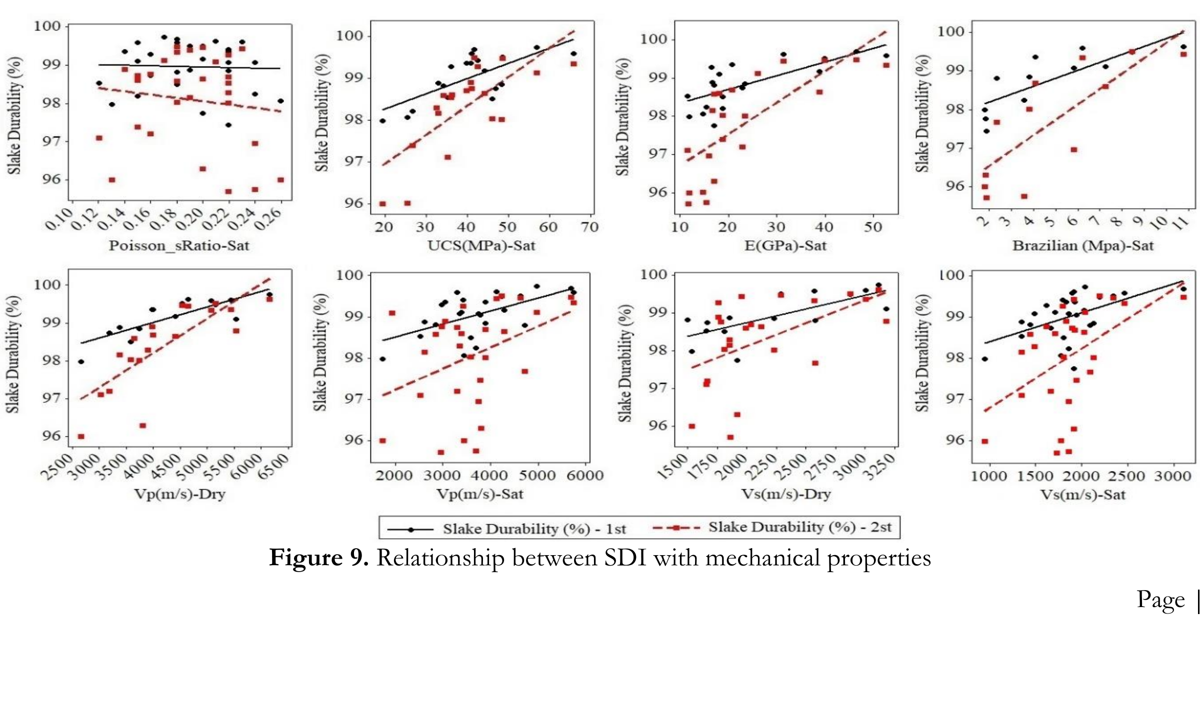

Figure 9. Relationship between SDI with mechanical properties

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The univariable relationship (95% Confidence intervals) between the SDI (cycles 1 and 2) and mechanical properties is shown in Fig. 9. As can be seen in this Fig., in the expected Tale Zang formation, with increased mechanical properties, SDI also increases. In this study, no significant relationship between SDI and υ (Sig> 0.05) was found (Table 2). Also, the relationship between SDI with $V_{p\text{-}sat}$, SDI_{2st} with $V_{s\text{-}dry}$, SDI with $V_{s\text{-}sat}$ are less accurate (R^2 <0.4). According to Table (2), in univariable linear regression, the use of BTS and $V_{p\text{-}dry}$ provides a more accurate prediction of SDI.

Table 2

Results of univariable regression analysis of mechanical properties

Regression equation	R square (R^2)	Result of ANOVA test	
		F	**Sig.**
$SDI_{1st} = 99.1 - 0.73\ \upsilon_{sat}$	0.00	0.05	0.83
$SDI_{2st} = 98.92 - 4.32\ \upsilon_{sat}$	0.02	0.44	0.51
$SDI_{1st} = 97.53 + 0.04\ UCS_{sat}$	0.50	17.09	0.00
$SDI_{2st} = 95.58 + 0.07\ UCS_{sat}$	0.53	20.30	0.00
$SDI_{1st} = 97.98 + 0.03E_{sat}$	0.52	18.53	0.00
$SDI_{2st} = 95.89 + 0.08\ E_{sat}$	0.55	23.25	0.00
$SDI_{1st} = 97.78 + 0.21\ BTS_{sat}$	0.65	18.44	0.00
$SDI_{2st} = 95.73 + 0.4\ BTS_{sat}$	0.62	16.22	0.00
$SDI_{1st} = 97.38 + 4.11 * 10^{-4}\ V_{p-dry}$	0.67	26.3	0.00
$SDI_{2st} = 94.57 + 9.11 * 10^{-4} V_{p-dry}$	0.63	28.78	0.00
$SDI_{1st} = 97.87 + 3.2 * 10^{-4} V_{p-sat}$	0.35	12.94	0.00
$SDI_{2st} = 96.21 + 5.13 * 10^{-4} V_{p-sat}$	0.15	4.97	0.03
$SDI_{1st} = 97.3 + 7.25 * 10^{-4} V_{s-dry}$	0.53	14.85	0.00
$SDI_{2st} = 95.67 + 12.22 * 10^{-4} V_{s-dry}$	0.28	8.02	0.01
$SDI_{1st} = 97.7 + 7.01 * 10^{-4} V_{s-sat}$	0.27	9.19	0.00
$SDI_{2st} = 95.36 + 14.36 * 10^{-4} V_{s-sat}$	0.22	7.17	0.01

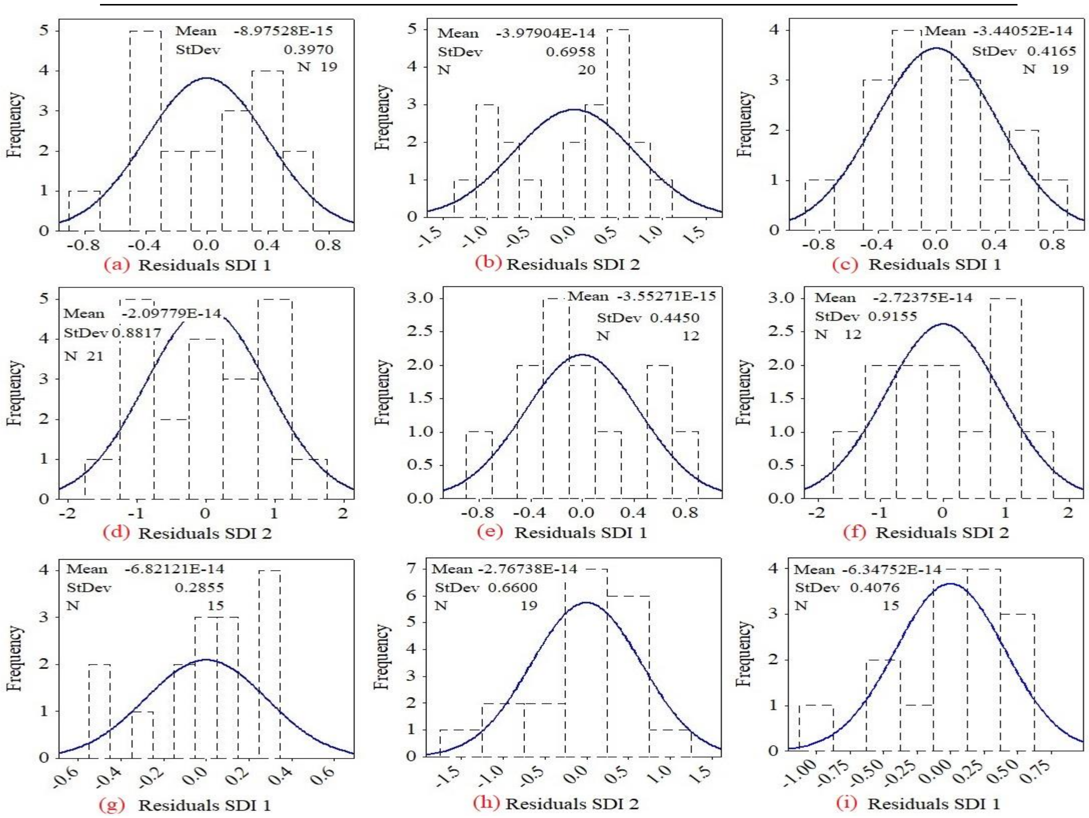

Figure 10. Frequency histogram and normal curve of univariable regression residuals mechanical properties. Relationship between: (a) SDI-1st with UCS, (b) SDI-2st with UCS, (c) SDI-1st with E, (d) SDI-2st with E, (e) SDI-1st with BTS, (f) SDI-2st with BTS, (g) SDI-1st with $V_{p\text{-}dry}$, (h) SDI-2st with $V_{p\text{-}dry}$ and (i) SDI-1st with $V_{s\text{-}dry}$

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According to Fig. 10, the mean of univariable regression residues of mechanical properties tends to zero. Also, the residuals frequency distribution function is normal in all cases (Fig. 10). The correlation between the measured values of SDI and the predicted values of univariable regression of mechanical properties is more than 0.89 (Fig. 11). Therefore, the analysis of residuals and graphs of predicted values versus measured values of SDI show high accuracy of regression relationships.

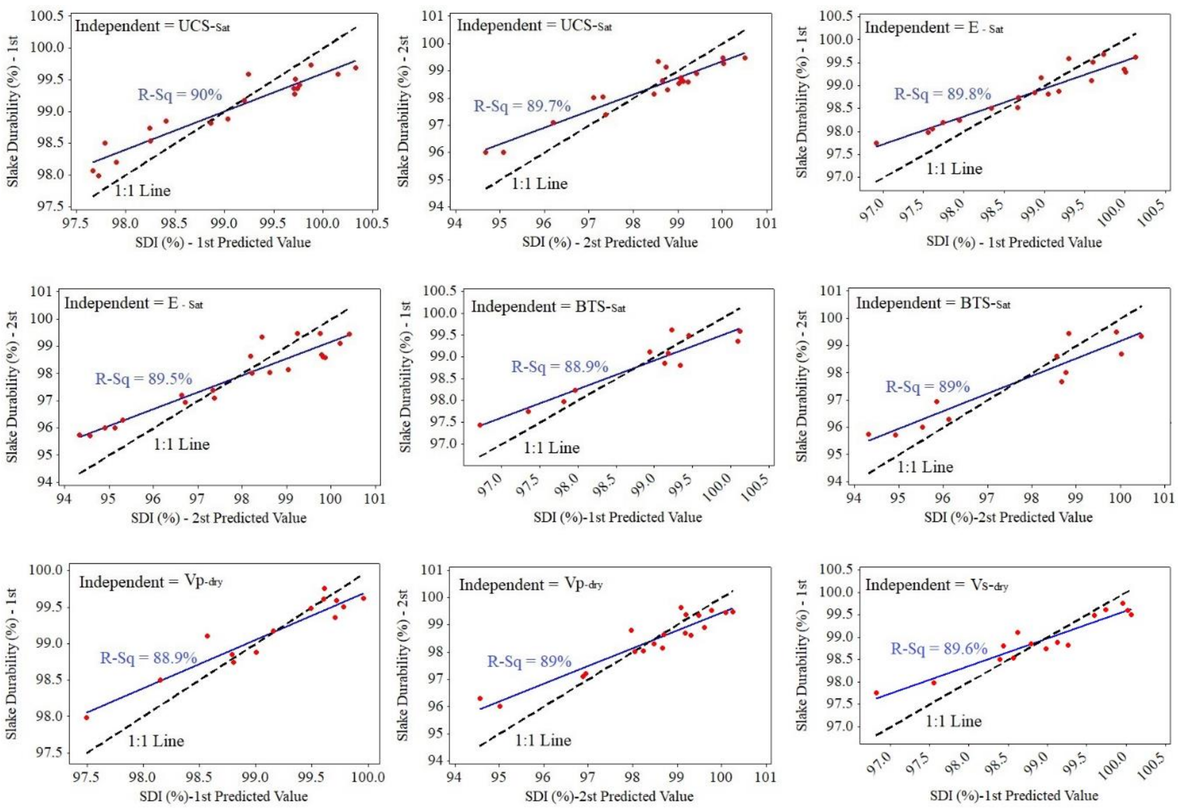

Figure 11. Comparison of measured and predicted SDI values using univariable regression of mechanical properties

Table 3
Results of analysis of the best two-variable regression relationships for predicting SDI using physical properties

Regression equation	R square (R^2)	Result of ANOVA test	
		F	Sig.
$\mathrm{SDI_{1st}} = 92.1 + 2.99\,\rho_{dry} - 0.075\,n$	0.66	33.33	0.00
$\mathrm{SDI_{2st}} = 81.4 + 7.1\,\rho_{dry} - 0.13\,n$	0.70	41.34	0.00
$\mathrm{SDI_{2st}} = 81.7 + 6.98\,\rho_{sat} - 0.21\,n$	0.70	41.17	0.00
$\mathrm{SDI_{2st}} = 99.8 - 2.67\mathrm{W} + 0.83\ \mathrm{n}$	0.66	35.52	0.00
$\mathrm{SDI_{2st}} = 82.3 + 6.72\rho_{dry} - 0.33W$	0.70	41.57	0.00
$\mathrm{SDI_{2st}} = 82.9 + 6.49\rho_{sat} - 0.50W$	0.70	41.53	0.00

Prediction of Slake Durability Index (SDI) using multivariable linear regression

Multivariable regression analysis is used to obtain the best relationship between variables when there is more than one input parameter. In general, the main issue of the multivariable regression method is to produce a relationship between input and output parameters. presents the two-variable regression analysis of physical properties for SDI prediction. As can be seen in this

Multivariable linear regression is generally expressed as the relationship between the output or dependent variables and input or independent variables [34]. Several factors affect the SDI value and the use of multivariable regression can show the role of each of these factors in the predicted SDI values. Table (3)

Table, the accuracy of these relationships concerning univariable regression shows a significant increase. In two-variable regression, SDI_{2st} shows more correlation with physical properties than SDI_{1st}.

The average of the residuals of these relationships tends to zero (Figure 12) and for all relationships, the residuals of zero have a maximum frequency. One-to-one diagrams of these relationships with $R^2 > 0.889$ also confirm the high accuracy of these relationships (Fig. 13).

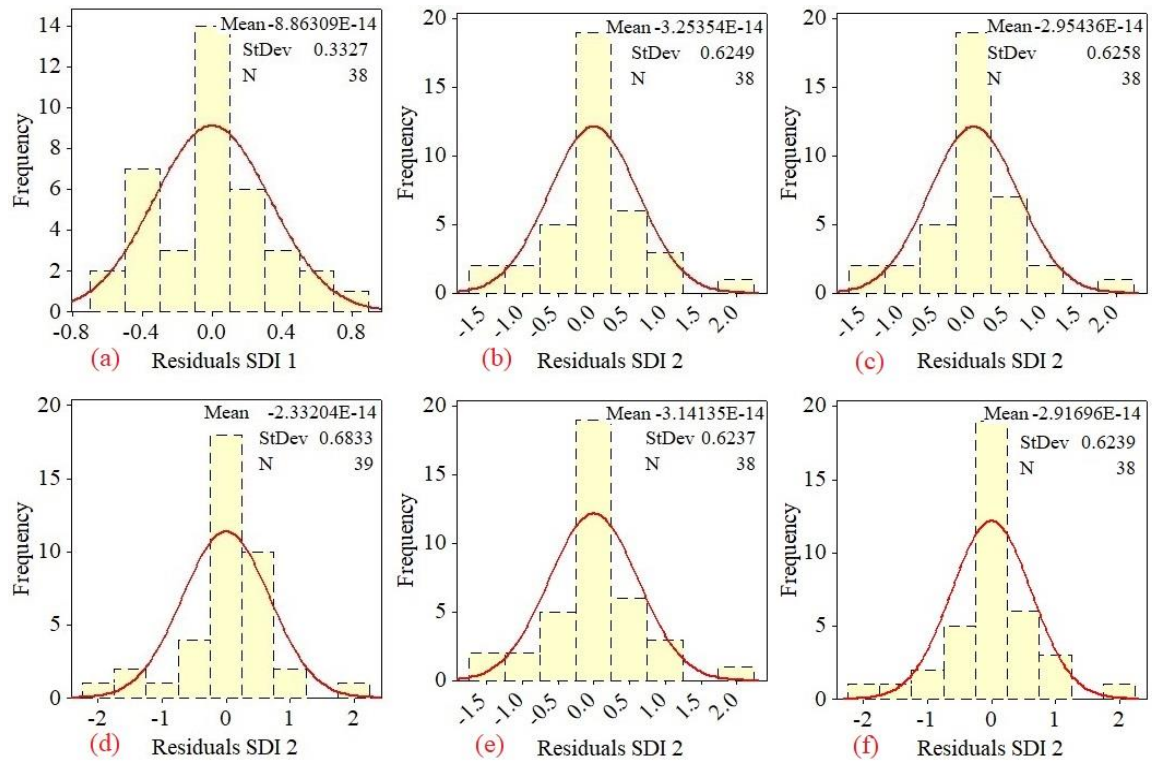

Figure 12. Frequency histogram and normal curve of two-variable regression residuals physical properties. Relationship between: (a) SDI-1st with n and ϱ-dry, (b) SDI-2st with n and ϱ-dry, (c) SDI-2st with n and ϱ-sat, (d) SDI-2st with W and n, (e) SDI-2st with W and ϱ-dry and (f) SDI-2st with W and ϱ-sat

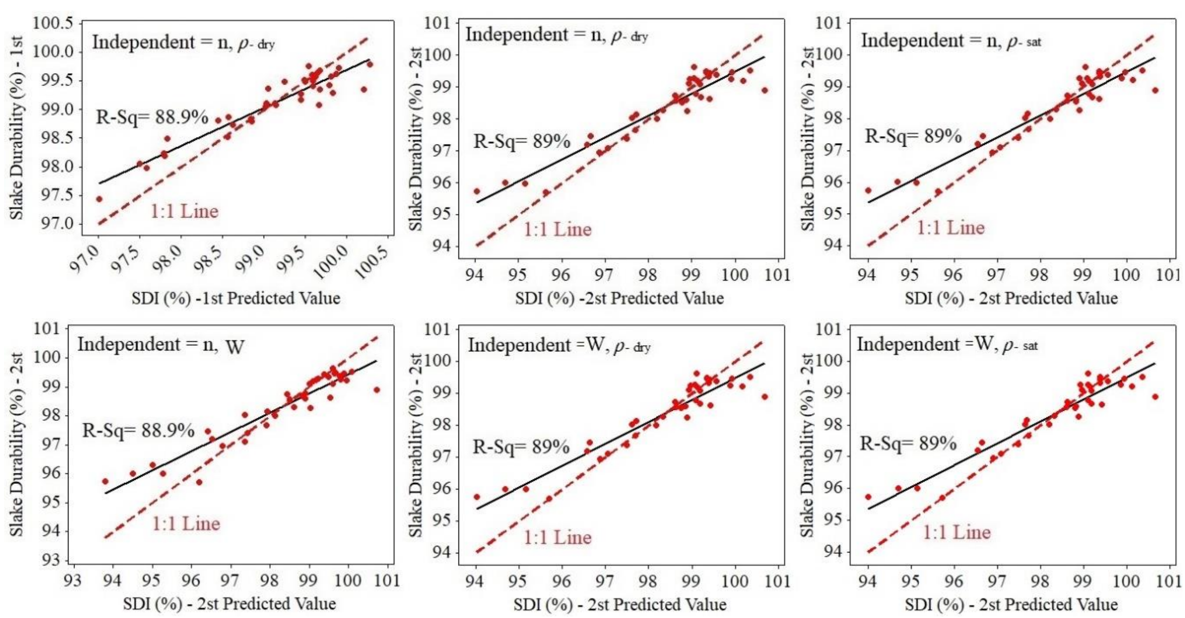

Figure 13. Comparison of measured and predicted SDI values using two-variable regression of physical properties

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Table (4) presents the results of two-variate regression analysis of mechanical properties. According to this table, using V_{p-sat} and BTS_{sat} to predict SDI_{1st} and using BTS_{sat} and E_{sat} to predict SDI_{2st} is the most accurate.

Table 4
Results of analysis of the best two-variable regression relationships for predicting SDI using mechanical properties

Regression equation	R square (R²)	Result of ANOVA test	
		F	Sig.
$SDI_{1st} = 97.7 + 0.01UCS_{sat} + 0.03E_{sat}$	0.72	13.99	0.00
$SDI_{2st} = 94.9 + 0.06E_{sat} + 0.28BTS_{sat}$	0.77	11.74	0.01
$SDI_{1st} = 97.3 + 0.12BTS_{sat} + 2.76 * 10^{-4}V_{p-sat}$	0.79	9.6	0.02
$SDI_{2st} = 93.9 + 0.3BTS_{sat} + 5.93 * 10^{-4}V_{p-sat}$	0.70	9.71	0.01
$SDI_{1st} = 97 + 0.16BTS_{sat} + 5.85 * 10^{-4}V_{s-sat}$	0.71	7.39	0.02
$SDI_{2st} = 93.5 + 0.33BTS_{sat} + 1.37 * 10^{-3}V_{s-sat}$	0.68	8.36	0.01

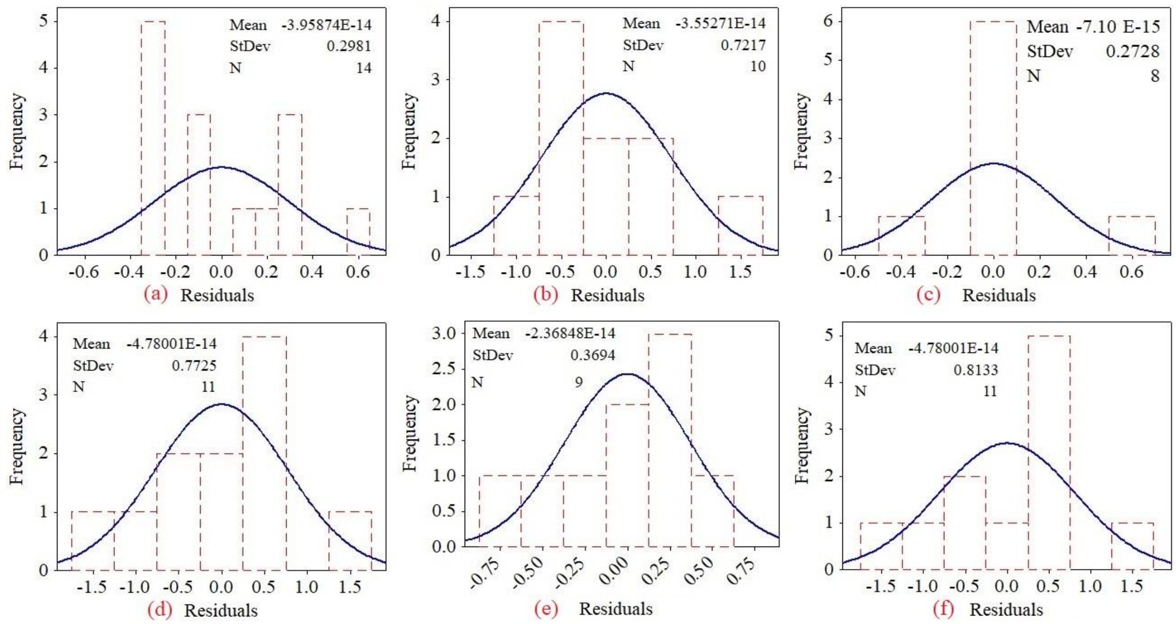

Figure 14. Frequency histogram and normal curve of two-variable regression residuals mechanical properties. Relationship between: (a) SDI-1st with UCS and E, (b) SDI-2st with BTS and E, (c) SDI-1st with BTS and V_{p-sat}, (d) SDI-2st with BTS and V_{p-sat}, (e) SDI-1st with BTS and V_{s-sat} and (f) SDI-2st with BTS and V_{s-sat}

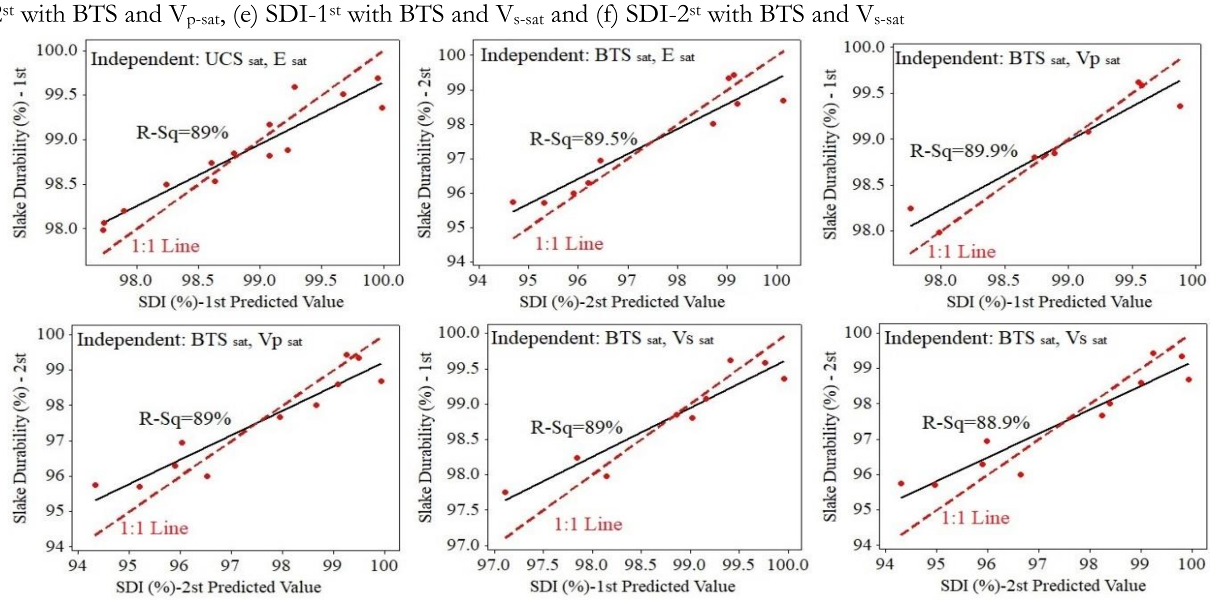

Figure 15. Comparison of measured and predicted SDI values using two-variable regression of mechanical properties

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Fig. 14 presents the residual analysis of two-variable regression relations of mechanical properties and SDI. For all relationships, the mean of the residuals is zero. Also, the distribution of residuals is almost normal for all relationships. As shown in Fig. 14c, for BTS and $V_{p\text{-}sat}$ regression with SDI_{1st}, the frequency of zero residues is maximal and shows the highest accuracy. The correlation between the measured and predicted values of SDI using two-variate regression of mechanical properties is more than 0.88, so the accuracy of predicted values of SDI is high (Fig. 15).

In order to determine the role of each physical and mechanical property in SDI prediction, three-variable, and higher regression was used. The best three-variable regression relationships of physical properties with SDI are shown in Table (5). According to this Table, the correlation between SDI_{2st} is higher compared to SDI_{1st} with physical properties. Fig. 16 shows the frequency of residues and Fig. 17 shows the measured and predicted values of SDI using these equations. According to Fig. 16, for all relations, the mean of the residuals tends to zero, and also the frequency of residuals is zero. The analysis of the residuals of the three-variable physical properties regression shows a higher accuracy of these relationships than the two-variable regression of the physical properties. There is a correlation between measured and predicted values using three-variable physical properties regression with SDI of more than 0.88 (Fig. 17).

Table 5
Results of analysis of the best three-variable regression relationships for predicting SDI using physical properties

Regression equation	R square (R^2)	Result of ANOVA test	
		F	Sig.
$SDI_{1st} = 95.1 + 1.78\ \rho_{sat} + 0.39\ n - 1.2\ W$	0.67	22.62	0.00
$SDI_{2st} = 84 + 6.03\ \rho_{sat} + 0.22\ n - 1.04\ W$	0.70	26.98	0.00
$SDI_{1st} = 94.7 + 1.92\ \rho_{dry} + 0.39\ n - 1.14\ W$	0.67	22.76	0.00
$SDI_{2st} = 83.6 + 6.2\ \rho_{dry} + 0.25\ n - 0.95\ W$	0.70	27.04	0.00

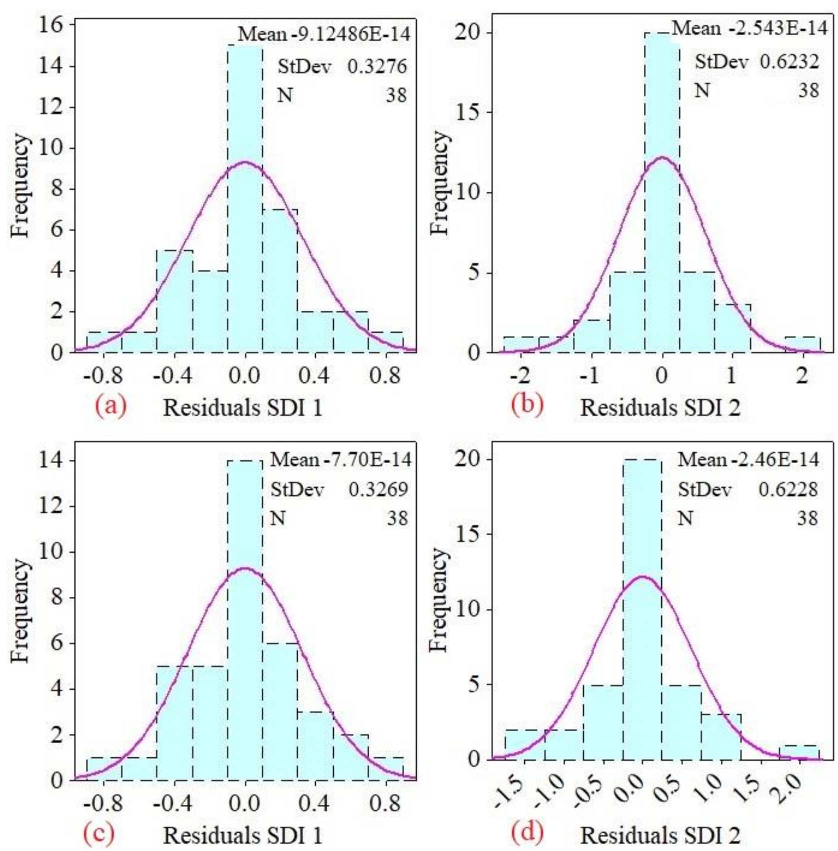

Figure 16. Frequency histogram and normal curve of three-variable regression residuals physical properties. Relationship between: (a) SDI-1st with n, W and ϱ- sat, (b) SDI-2st with n, W and ϱ- sat, (c) SDI-1st with n, W and ϱ-dry and (d) SDI-2st with n, W and ϱ-dry

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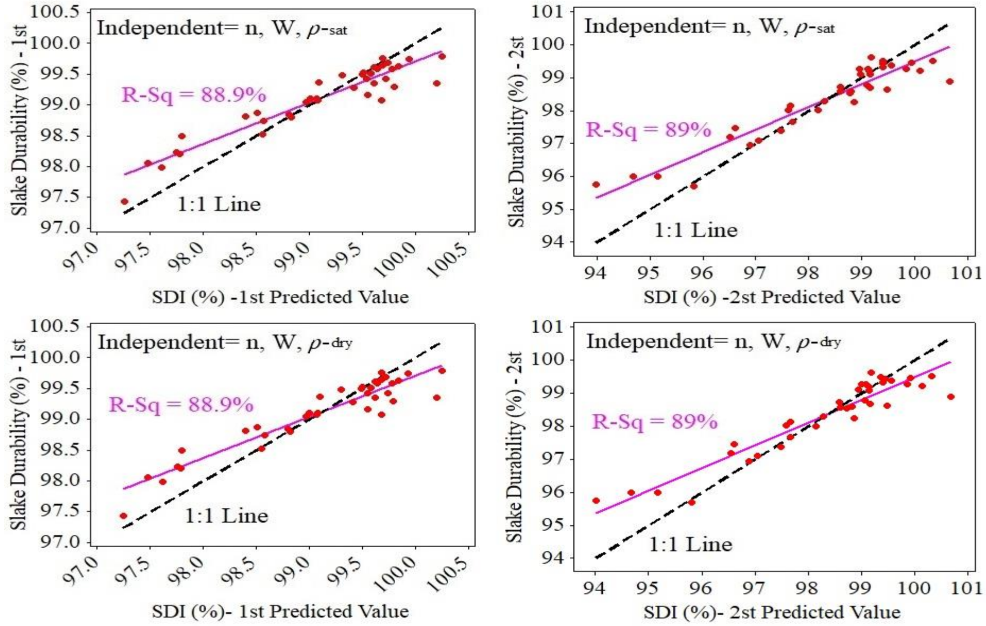

Figure 17. Comparison of measured and predicted SDI values using three-variable regression of physical properties

Table 6
Results of analysis of the best three-variable regression relationships for predicting SDI using mechanical properties

Regression equation	R square (R^2)	Result of ANOVA test	
		F	Sig.
$\mathrm{SDI}_{1st} = 97.7 + 0.01UCS_{sat} + 0.02E_{sat} + 1.28 * 10^{-4}V_{P-sat}$	0.72	7.91	0.01
$\mathrm{SDI}_{1st} = 97.6 + 0.01UCS_{sat} + 0.02E_{sat} + 2.63 * 10^{-4}V_{S-sat}$	0.73	8.32	0.01
$\mathrm{SDI}_{2st} = 95.2 + 0.04UCS_{sat} + 0.02E_{sat} + 4.49 * 10^{-4}V_{S-sat}$	0.64	5.24	0.02
$\mathrm{SDI}_{2st} = 95 + 0.06E_{sat} + 0.28BTS_{sat} - 3.6 * 10^{-4}V_{P-sat}$	0.77	6.71	0.02
$\mathrm{SDI}_{2st} = 94.7 + 0.05E_{sat} + 0.28BTS_{sat} + 1.8 * 10^{-4}V_{S-sat}$	0.77	6.75	0.02
$\mathrm{SDI}_{2st} = 94.3 + 0.3BTS_{sat} + 8.9 * 10^{-4}V_{P-sat} - 8.2 * 10^{-4}V_{S-sat}$	0.71	5.79	0.03

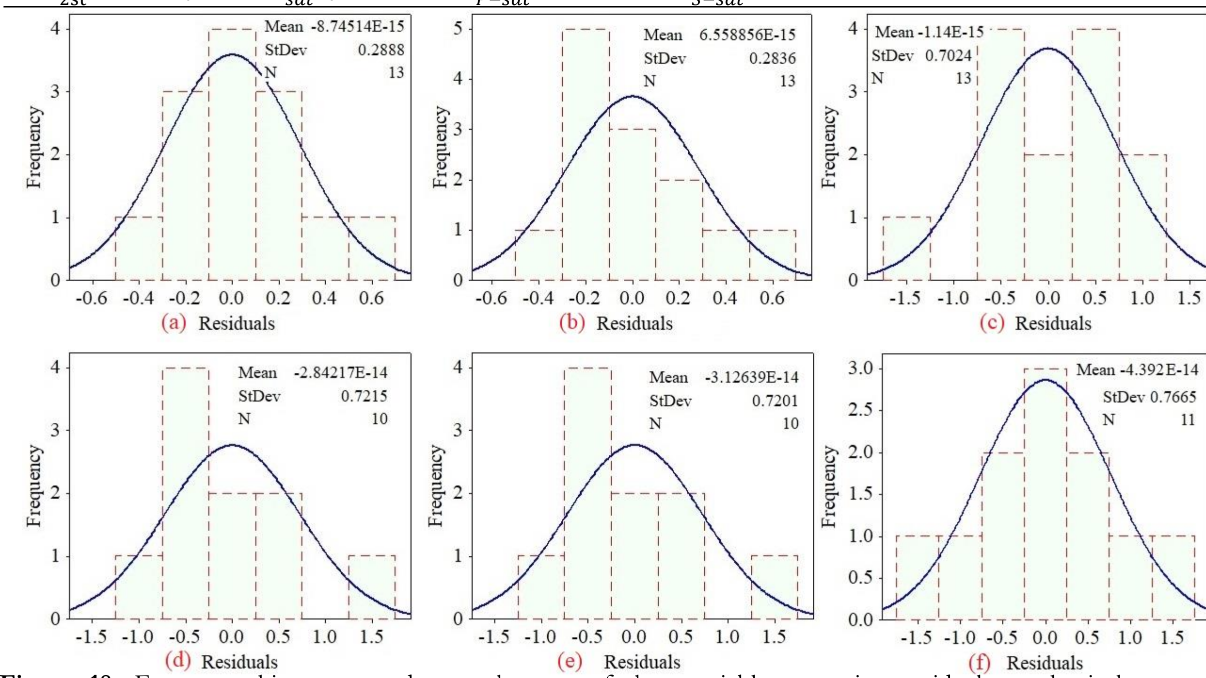

Figure 18. Frequency histogram and normal curve of three-variable regression residuals mechanical properties. Relationship between: (a) SDI-1st with UCS, E and V_{p-sat}, (b) SDI-1st with UCS, E and V_{s-sat}, (c) SDI-2st with UCS, E and V_{s-sat}, (d) SDI-2st with BTS, E and V_{p-sat}, (e) SDI-2st with BTS, E and V_{s-sat} and (f) SDI-2st with BTS, V_{s-sat} and V_{p-sat}

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Similarly, Table (6) shows the best three-variable regression relationships of mechanical properties with SDI. As can be seen in this Fig., the SDI_{2st} prediction has more accurate results using mechanical properties. The results show that with increasing the number of variables, a more accurate prediction of SDI_{2st} compared to SDI_{1st} is made. The analysis of the residuals of these relationships (Fig. 18) and the degree of correlation between the measured and predicted SDI values (Fig. 19) show the high validity of these relationships.

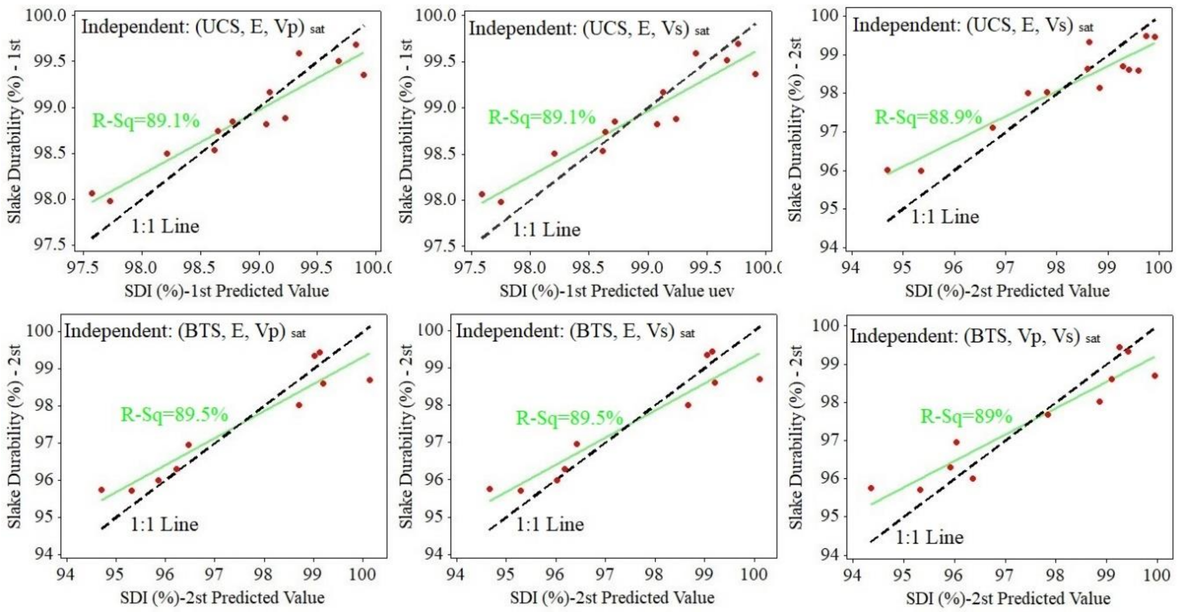

Figure 19. Comparison of measured and predicted SDI values using three-variable regression of mechanical properties

Discussion

In this study, as expected, SDI decreased with increasing porosity (n) and W. SDI also increases with increasing ϱ, which is confirmed by studies by other researchers (Table 7). According to studies, with increased mechanical properties, SDI also increases, which is consistent with the results of other researchers (Table 7). The results show that with increasing the number of test cycles, SDI decreases with different ratios for different samples (Fig. 20). In fact, with increasing number of cycles, the durability index of the samples has decreased due to the process of abrasion and wetting and drying. Also, with increasing the number of wetting and drying cycles of the slake durability test, its correlation coefficient with physical and mechanical properties increases in most cases (Fig. 21). This indicates that in the first cycle, due to the angularity of the specimens, the weight loss that occurs during the test indicates the actual durability of the specimen. Increasing the number of cycles due to the elimination of roughness and corners of the samples will achieve the actual durability. As a result, the first cycle test has little validity for a more accurate assessment of durability. Similarly, in the study of Gokceoglu et al. (2000), with increasing the number of cycles of SDI test, its correlation coefficient with uniaxial compressive strength increased. Results of Gamble [23] studies; Ulusay et al. [35]; Nikudel et al. [36]; Fereidooni and Khajevand [6] and Seyed Mousavi et al. [11] also showed that the SDI test results provide more realistic results in several cycles.

Table 7
Comparison of the results of this study with previous studies

Regression equation	R square (R^2)	References
$SDI_{1st} = 99.98 - 0.15\ n$	0.60	This study
$SDI_{2st} = 100.1 - 0.31\ n$	0.64	
$SDI_{1st} = 86.22 + 5.15\rho_{dry}$	0.59	
$SDI_{2st} = 70.53 + 11.12\rho_{dry}$	0.62	
$SDI_{1st} = 81.74 + 6.78\ \rho_{sat}$	0.48	
$SDI_{2st} = 60.59 + 14.73\ \rho_{sat}$	0.52	
$SDI_{1st} = 92.94 - 0.37\ W$	0.61	
$SDI_{2st} = 100.1 - 0.74\ W$	0.65	

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$\rho_{dry} = 0.88\ e^{0.02\ SDI_{2st}}$	0.77	Nikudel et al. [37]
$\rho_{sat} = 0.66\ e^{0.01\ SDI_{2st}}$	0.73	
$n = -179.1\ \mathrm{Ln}\ (SDI_{2st}) + 829.11$	0.71	
$UCS = (3 \times 10^{-0.8}) \times\ e^{0.21\ SDI_{2st}}$	0.80	
$V_p = 24.51 e^{0.05\ SDI_{2st}}$	0.82	
$SDI_{1st} = 97.53 + 0.04\ UCS_{sat}$	0.50	This study
$SDI_{2st} = 95.58 + 0.07\ UCS_{sat}$	0.53	
$SDI_{1st} = 97.98 + 0.03 E_{sat}$	0.52	
$SDI_{2st} = 95.89 + 0.08\ E_{sat}$	0.55	
$SDI_{1st} = 97.78 + 0.21\ BTS_{sat}$	0.65	
$SDI_{2st} = 95.73 + 0.4\ BTS_{sat}$	0.62	
$SDI_{1st} = 97.38 + 4.11 * 10^{-4}\ V_{p-dry}$	0.67	
$SDI_{2st} = 94.57 + 9.11 * 10^{-4} V_{p-dry}$	0.63	
$SDI_{1st} = 97.3 + 7.25 * 10^{-4} V_{s-dry}$	0.53	
$SDI_{2st} = -13.8 + 1.52 * 10^{-3}\ UCS\ (KPa)$	0.63	Engin et al. [38]
$SDI_{2st} = 98.47 + 0.013\ UCS_{sat}$	0.94	Bashirgonbadi et al. [39]
$SDI_{15st} = 94.03 + 0.04\ UCS$	0.77	Nikudel et al. [36]
$SDI = 75.7 + 0.0043 V_p$	0.63	Abd El Aal and Kahraman [8]
$UCS = 17.8\ e^{0.0083\ SDI_{2st}}$	0.62	Arman [15]
$SDI_{1st} = 89.86 + 4.05 \rho_{dry}$	0.67	Fereidooni and Khajevand [6]
$SDI_{2st} = 89.74 + 3.85 \rho_{dry}$	0.72	
$SDI_{1st} = 91.1 + 3.4 \rho_{sat}$	0.55	
$SDI_{2st} = 88.89 + 4.15 \rho_{sat}$	0.67	
$SDI_{1st} = 100.26 - 0.28\ n$	0.74	
$SDI_{2st} = 100.26 - 0.31\ n$	0.79	
$SDI_{1st} = 100.03 - 0.65\ W$	0.81	
$SDI_{2st} = 99.67 - 0.74\ W$	0.85	
$SDI_{1st} = 97.09 + 0.0004\ V_p$	0.72	
$SDI_{2st} = 96.34 + 0.0004 V_p$	0.77	
$SDI_{1st} = 97.27 + 0.26 BTS$	0.78	
$SDI_{2st} = 96.6 + 0.3\ BTS$	0.83	
$SDI_{1st} = 97.05 + 0.07\ UCS$	0.91	
$SDI_{2st} = 96.62 + 0.07\ UCS$	0.92	

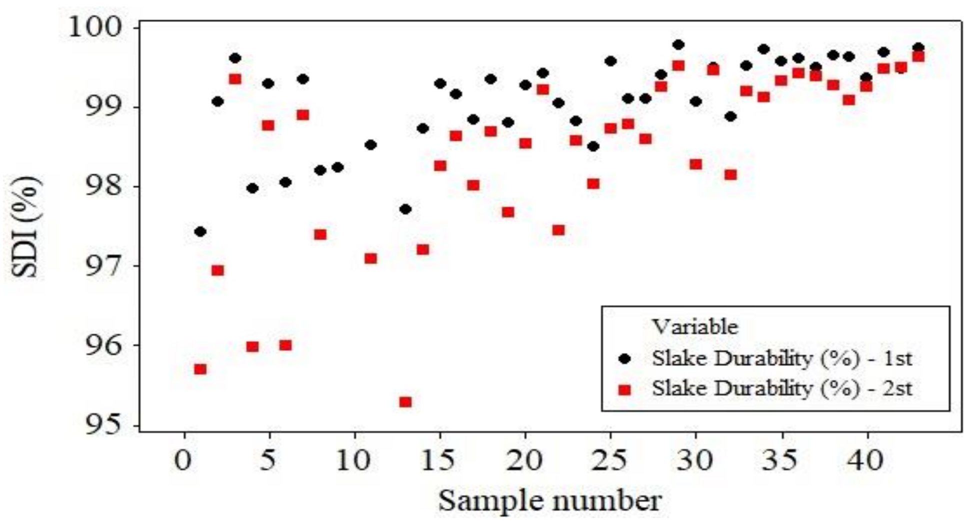

Figure 20. The SDI changes in the first and second cycles

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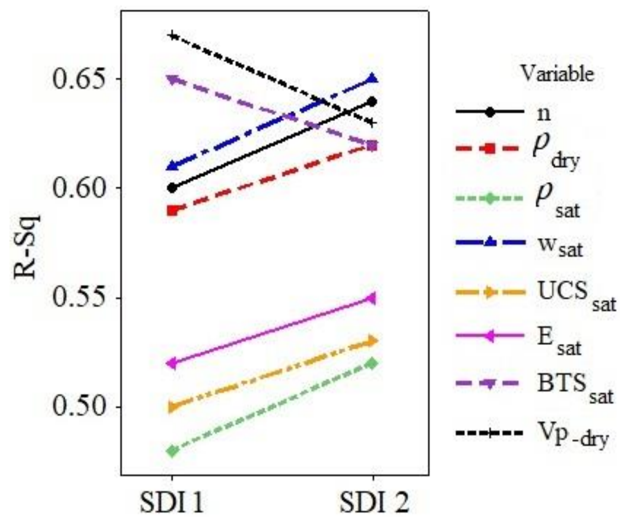

Figure 21. R square (R^2) changes in the first and second cycles of the SDI (Univariable regression)

Comparison of univariate and multivariate regression results shows that the use of SDI multivariate regression provides more reliable results (Fig. 22). In general, it can be stated that in the Tale Zang formation there is a higher correlation between SDI and mechanical properties (Fig. 22).

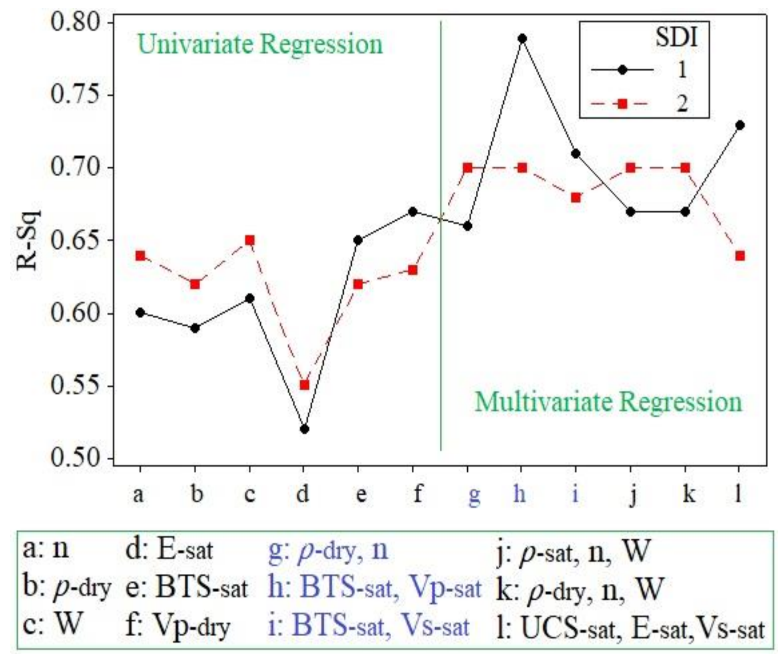

Figure 22. Comparison of R square (R^2) changes in univariable regression with multivariable regression

The use of multivariate SDI regression also allows the effect of different properties to be considered simultaneously. Using multivariate regression analysis, the property that has the most impact can be identified. Beta factor (β) is used for this purpose. The value of β for each of the multivariate SDI regressions is shown in Table (8). Any property with a larger β is more important in the regression model. As can be seen in Table (8), the properties n, UCS, E, and BTS in different relationships have the greatest impact on the SDI value.

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Table 8
Beta values for three-variable regression equations

Regression equation	Variables	Standardize Coefficients Beta
$SDI_{1st} = 95.1 + 1.78\ \rho_{sat} + 0.39\ n - 1.2\ W$	ρ_{sat}	0.184
	n	2.073
	W	-2.728
$SDI_{2st} = 84 + 6.03\ \rho_{sat} + 0.22\ n - 1.04\ W$	ρ_{sat}	0.308
	n	0.592
	W	-1.176
$SDI_{1st} = 94.7 + 1.92\ \rho_{dry} + 0.39\ n - 1.14\ W$	ρ_{dry}	0.282
	n	2.059
	W	-2.597
$SDI_{2st} = 83.6 + 6.2\ \rho_{dry} + 0.25\ n - 0.95\ W$	ρ_{dry}	0.451
	n	0.669
	W	-1.077
$SDI_{1st} = 97.7 + 0.01UCS_{sat} + 0.02E_{sat} + 1.28 * 10^{-4}V_{P-sat}$	UCS_{sat}	0.147
	E_{sat}	0.471
	V_{p-sat}	0.275
$SDI_{1st} = 97.6 + 0.01UCS_{sat} + 0.02E_{sat} + 2.63 * 10^{-4}V_{S-sat}$	UCS_{sat}	0.186
	E_{sat}	0.466
	V_{s-sat}	0.270
$SDI_{2st} = 95.2 + 0.04UCS_{sat} + 0.02E_{sat} + 4.49 * 10^{-4}V_{S-sat}$	UCS_{sat}	0.415
	E_{sat}	0.235
	V_{s-sat}	0.218
$SDI_{2st} = 95 + 0.06E_{sat} + 0.28BTS_{sat} - 3.6 * 10^{-4}V_{P-sat}$	E_{sat}	0.483
	BTS_{sat}	0.543
	V_{p-sat}	-0.024
$SDI_{2st} = 94.7 + 0.05E_{sat} + 0.28BTS_{sat} + 1.8 * 10^{-4}V_{S-sat}$	E_{sat}	0.429
	BTS_{sat}	0.544
	V_{s-sat}	0.046
$SDI_{2st} = 94.3 + 0.3BTS_{sat} + 8.9 * 10^{-4}V_{P-sat} - 8.2 * 10^{-4}V_{S-sat}$	BTS_{sat}	0.585
	V_{p-sat}	0.619
	V_{s-sat}	-0.212

Conclusion

In the present study, an experimental study was performed to predict SDI in the limestones of the Tale Zang Formation using different regression models. Indirect methods are widely used to estimate rock strength parameters. For this purpose, different physical and mechanical properties were used to predict SDI. The results showed that:

1- The studied samples are in a very resistant category based on different classifications of SDI, and as a result, the probability of a sharp decrease in their resistance to water is very low.

2- Considering that establishing correlations between the results of different engineering experiments, causes the replacement of simple relationships instead of complex ones and also accelerates the evaluation of the durability of stone materials. Experimental relationships between SDI and physical and mechanical properties were presented by examining the results obtained from various experiments. Given the univariate linear regression relationships, there is a direct relationship between SDI with ϱ, UCS, E, BTS, Vp, Vs, and an inverse relationship with n and W.

3- As the number of wetting and drying Cycles increases, SDI decreases.

4- SDI_{2st} offers relationships with higher correlation than SDI_{1st}, with physical and mechanical properties.

5- In general, as the number of wetting and drying cycles increases, the correlation coefficient between SDI and physical and mechanical properties increases. This indicates that the initial cycles due to the unevenness and angularity of the specimens, the weight loss that occurs during the SDI test, does not indicate the actual durability of the specimen and with an increasing number of cycles due to the disappearance

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of unevenness and angles the samples have real durability.

6- The use of multivariable regression provides a more accurate prediction of SDI. In multivariable regression models, n, UCS, E and BTS properties in different relationships have the greatest effect on the SDI value.

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